> restart

> $Ecua := diff(y(x,t),x) + diff(y(x,t),x,t) - diff(y(x,t),t) = 0$

$$Ecua := \frac{\partial}{\partial x} y(x,t) + \frac{\partial^2}{\partial t \partial x} y(x,t) - \frac{\partial}{\partial t} y(x,t) = 0 \quad \textbf{(1)}$$

> $EcuaDos := eval(subs(y(x,t) = P(x) \cdot Q(t), Ecua))$

$$EcuaDos := \left(\frac{d}{dx} P(x)\right) Q(t) + \left(\frac{d}{dx} P(x)\right)\left(\frac{d}{dt} Q(t)\right) - P(x)\left(\frac{d}{dt} Q(t)\right) = 0 \quad \textbf{(2)}$$

> $EcuaSepUno := lhs(EcuaDos) + P(x)\left(\frac{d}{dt} Q(t)\right) = rhs(EcuaDos) + P(x)\left(\frac{d}{dt} Q(t)\right)$

$$EcuaSepUno := \left(\frac{d}{dx} P(x)\right) Q(t) + \left(\frac{d}{dx} P(x)\right)\left(\frac{d}{dt} Q(t)\right) = P(x)\left(\frac{d}{dt} Q(t)\right) \quad \textbf{(3)}$$

> $EcuaSepFinal := simplify\left(\frac{(lhs(EcuaSepUno))}{P(x) \cdot (Q(t) + diff(Q(t),t))}\right) = \frac{(rhs(EcuaSepUno))}{P(x) \cdot (Q(t) + diff(Q(t),t))}$

$$EcuaSepFinal := \frac{\frac{d}{dx} P(x)}{P(x)} = \frac{\frac{d}{dt} Q(t)}{Q(t) + \frac{d}{dt} Q(t)} \quad \textbf{(4)}$$

> $EcuaX := lhs(EcuaSepFinal) = \mathrm{alpha}$

$$EcuaX := \frac{\frac{d}{dx} P(x)}{P(x)} = \alpha \quad \textbf{(5)}$$

> $EcuaT := rhs(EcuaSepFinal) = \mathrm{alpha}$

$$EcuaT := \frac{\frac{d}{dt} Q(t)}{Q(t) + \frac{d}{dt} Q(t)} = \alpha \quad \textbf{(6)}$$

> $EcuaXcero := subs(\mathrm{alpha} = 0, EcuaX)$

$$EcuaXcero := \frac{\frac{d}{dx} P(x)}{P(x)} = 0 \quad \textbf{(7)}$$

> $EcuaTcero := subs(\mathrm{alpha} = 0, EcuaT)$

$$EcuaTcero := \frac{\frac{d}{dt} Q(t)}{Q(t) + \frac{d}{dt} Q(t)} = 0 \quad \textbf{(8)}$$

> $SolXcero := dsolve(EcuaXcero)$

$$SolXcero := P(x) = c_1 \quad \textbf{(9)}$$

> $SolTcero := dsolve(EcuaTcero)$

$$SolTcero := Q(t) = c_1 \quad \textbf{(10)}$$

> $SolGralCero := y(x,t) = \_C1$

$$SolGralCero := y(x,t) = c_1 \quad \textbf{(11)}$$

> $EcuaXpos := subs(\text{alpha}=\beta^2, EcuaX)$

$$EcuaXpos := \frac{\frac{\mathrm{d}}{\mathrm{d}x} P(x)}{P(x)} = \beta^2 \tag{12}$$

> $EcuaTpos := subs(\text{alpha}=\beta^2, EcuaT)$

$$EcuaTpos := \frac{\frac{\mathrm{d}}{\mathrm{d}t} Q(t)}{Q(t) + \frac{\mathrm{d}}{\mathrm{d}t} Q(t)} = \beta^2 \tag{13}$$

> $SolXpos := dsolve(EcuaXpos)$

$$SolXpos := P(x) = c_1 \, \mathrm{e}^{\beta^2 x} \tag{14}$$

> $SolTpos := simplify(dsolve(EcuaTpos))$

$$SolTpos := Q(t) = c_1 \, \mathrm{e}^{-\frac{\beta^2 t}{(\beta-1)(\beta+1)}} \tag{15}$$

> $SolGralPos := y(x,t) = (subs(c_1 = 1, rhs(SolXpos))) \cdot (rhs(SolTpos))$

$$SolGralPos := y(x,t) = \mathrm{e}^{\beta^2 x} c_1 \, \mathrm{e}^{-\frac{\beta^2 t}{(\beta-1)(\beta+1)}} \tag{16}$$

> $EcuaXneg := subs(\text{alpha}=-\beta^2, EcuaX)$

$$EcuaXneg := \frac{\frac{\mathrm{d}}{\mathrm{d}x} P(x)}{P(x)} = -\beta^2 \tag{17}$$

> $EcuaTneg := subs(\text{alpha}=-\beta^2, EcuaT)$

$$EcuaTneg := \frac{\frac{\mathrm{d}}{\mathrm{d}t} Q(t)}{Q(t) + \frac{\mathrm{d}}{\mathrm{d}t} Q(t)} = -\beta^2 \tag{18}$$

> $SolXneg := dsolve(EcuaXneg)$

$$SolXneg := P(x) = c_1 \, \mathrm{e}^{-\beta^2 x} \tag{19}$$

> $SolTneg := dsolve(EcuaTneg)$

$$SolTneg := Q(t) = c_1 \, \mathrm{e}^{-\frac{\beta^2 t}{\beta^2+1}} \tag{20}$$

> $SolGralNeg := y(x,t) = (subs(c_1 = 1, rhs(SolXneg))) \cdot (rhs(SolTneg))$

$$SolGralNeg := y(x,t) = \mathrm{e}^{-\beta^2 x} c_1 \, \mathrm{e}^{-\frac{\beta^2 t}{\beta^2+1}} \tag{21}$$

> $SolGralPos$

$$y(x,t) = \mathrm{e}^{\beta^2 x} c_1 \, \mathrm{e}^{-\frac{\beta^2 t}{(\beta-1)(\beta+1)}} \tag{22}$$

> $SolGralCero$

$$y(x,t)=c_1 \tag{23}$$

> $Ecua$

$$\frac{\partial}{\partial x}\,y(x,t)+\frac{\partial^2}{\partial t\,\partial x}\,y(x,t)-\frac{\partial}{\partial t}\,y(x,t)=0 \tag{24}$$

> $ComprobarUno := simplify(eval(subs(y(x,t)=rhs(SolGralCero), Ecua)))$

$$ComprobarUno := 0=0 \tag{25}$$

> $ComprobarDos := simplify(eval(subs(y(x,t)=rhs(SolGralPos), Ecua)))$

$$ComprobarDos := 0=0 \tag{26}$$

> $ComprobarTres := simplify(eval(subs(y(x,t)=rhs(SolGralNeg), Ecua)))$

$$ComprobarTres := 0=0 \tag{27}$$

> $EcuaDos$

$$\left(\frac{d}{dx}\,P(x)\right)Q(t)+\left(\frac{d}{dx}\,P(x)\right)\left(\frac{d}{dt}\,Q(t)\right)-P(x)\left(\frac{d}{dt}\,Q(t)\right)=0 \tag{28}$$

> $EcuaSepTres := lhs(EcuaDos)-\left(\left(\frac{d}{dx}\,P(x)\right)\left(\frac{d}{dt}\,Q(t)\right)-P(x)\left(\frac{d}{dt}\,Q(t)\right)\right)$
> $=rhs(EcuaDos)-\left(\left(\frac{d}{dx}\,P(x)\right)\left(\frac{d}{dt}\,Q(t)\right)-P(x)\left(\frac{d}{dt}\,Q(t)\right)\right)$

$$EcuaSepTres := \left(\frac{d}{dx}\,P(x)\right)Q(t)=-\left(\frac{d}{dx}\,P(x)\right)\left(\frac{d}{dt}\,Q(t)\right)+P(x)\left(\frac{d}{dt}\,Q(t)\right) \tag{29}$$

> $EcuaSepTresFinal := \frac{lhs(EcuaSepTres)}{Q(t)\cdot(P(x)-diff(P(x),x))}$
> $=simplify\left(\frac{rhs(EcuaSepTres)}{Q(t)\cdot(P(x)-diff(P(x),x))}\right)$

$$EcuaSepTresFinal := \frac{\frac{d}{dx}\,P(x)}{P(x)-\frac{d}{dx}\,P(x)}=\frac{\frac{d}{dt}\,Q(t)}{Q(t)} \tag{30}$$

> $EcuaSepFinal$

$$\frac{\frac{d}{dx}\,P(x)}{P(x)}=\frac{\frac{d}{dt}\,Q(t)}{Q(t)+\frac{d}{dt}\,Q(t)} \tag{31}$$

> $restart$

SERIE TRIGONOMÉTRICA DE FOURIER

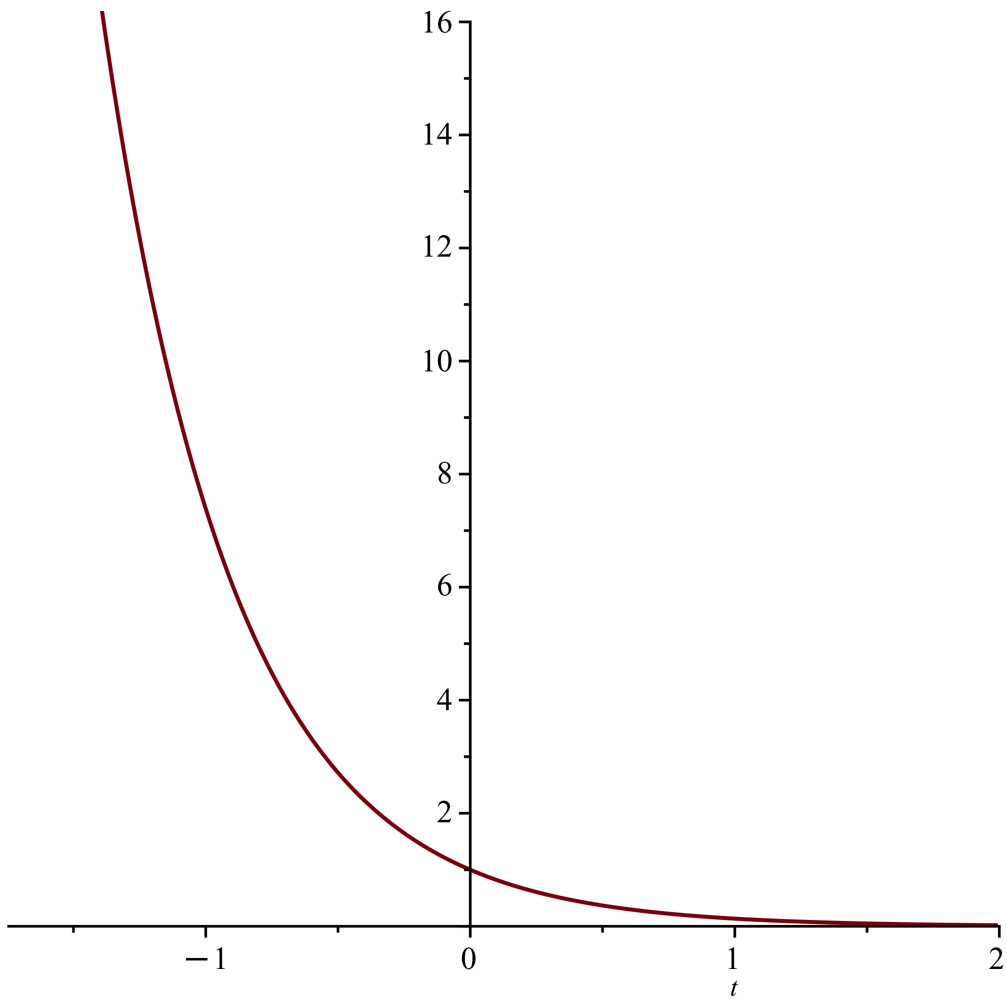
> $f := \exp(-2\cdot t)$

$$f := e^{-2t} \tag{32}$$

> $L := 2$

$$L := 2 \tag{33}$$

> $plot(f, t=-2..2)$



> $a[0] := \frac{1}{L} \cdot int(f, t = -L..L);\ evalf(\%, 3)$

$$a_0 := \frac{e^4}{4} - \frac{e^{-4}}{4}$$

$$13.6 \tag{34}$$

> $a[n] := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^n, \frac{1}{L} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t = -L..L\right)\right)$

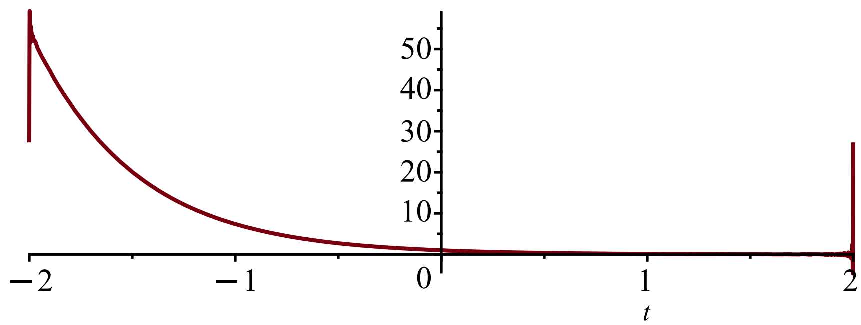
$$a_n := \frac{-4\, e^{-4} (-1)^n + 4\, e^4 (-1)^n}{n^2 \pi^2 + 16} \tag{35}$$

> $b[n] := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^n, \frac{1}{L} \cdot int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t = -L..L\right)\right)$

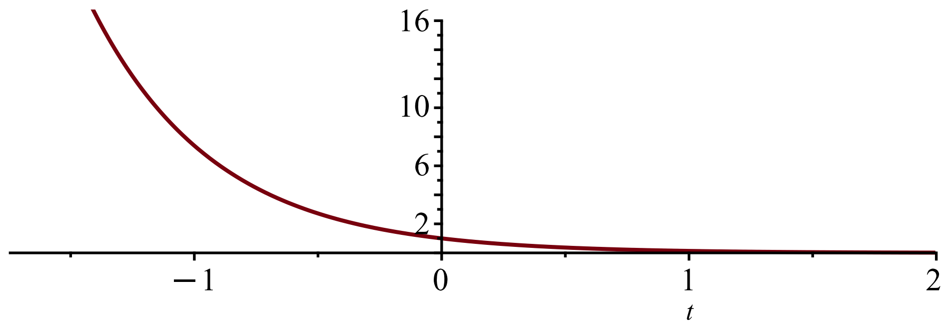
$$b_n := -\frac{e^{-4} (-1)^n \pi n - e^4 (-1)^n \pi n}{n^2 \pi^2 + 16} \tag{36}$$

> $ff := \frac{a[0]}{2} + sum\left(a[n] \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right) + b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), n = 1..1000\right):$

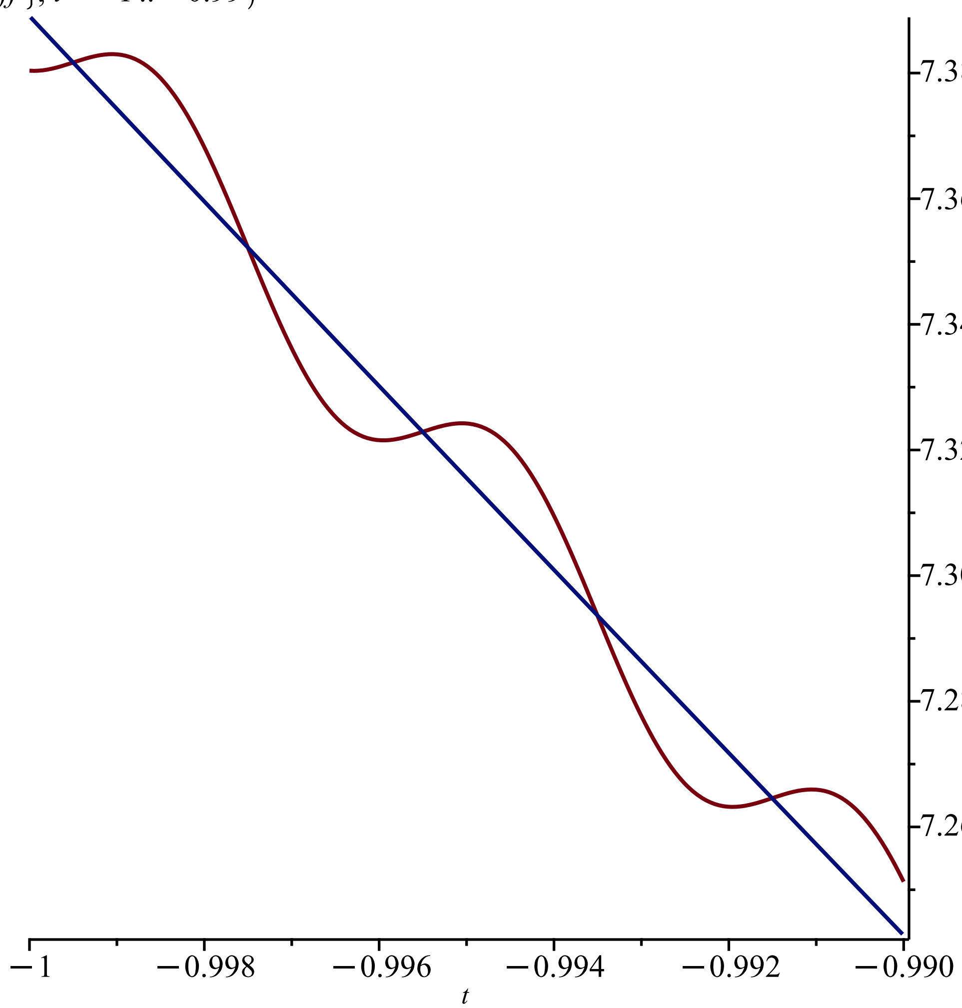
> $plot(ff, t = -L..L)$



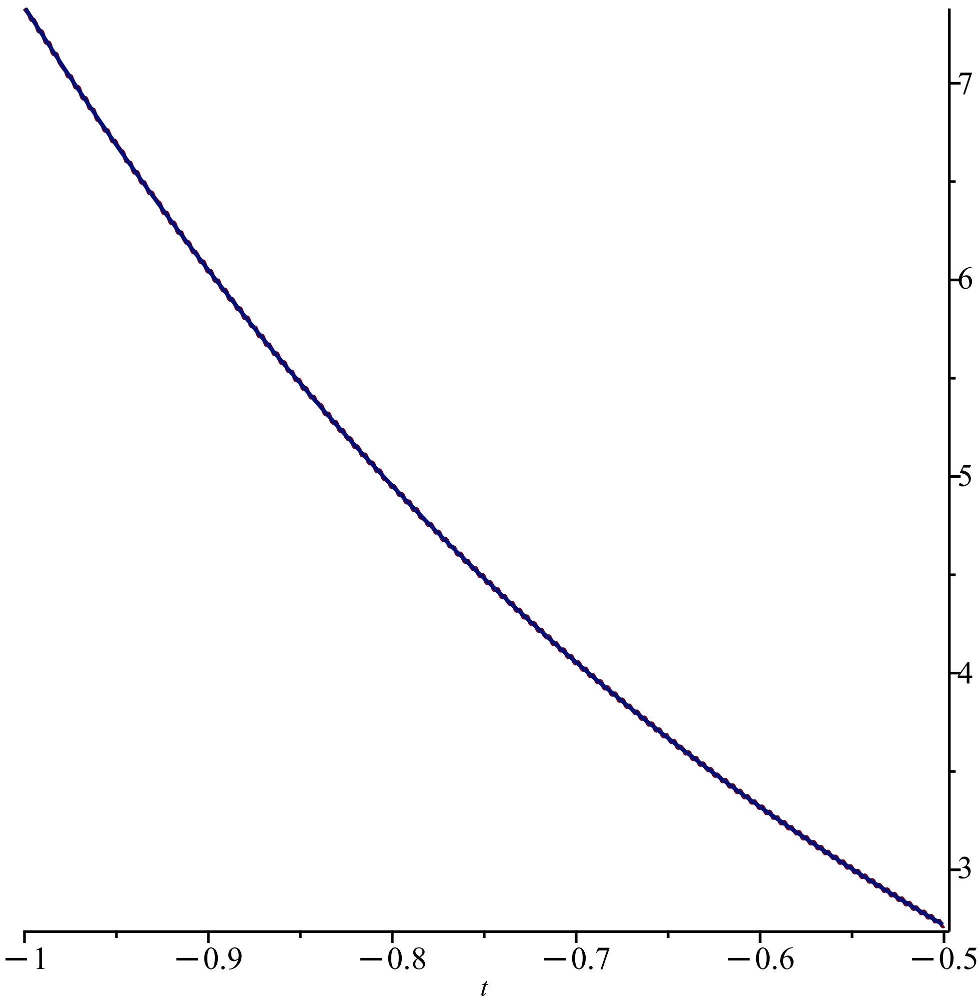
> $plot(f, t=-2..2)$



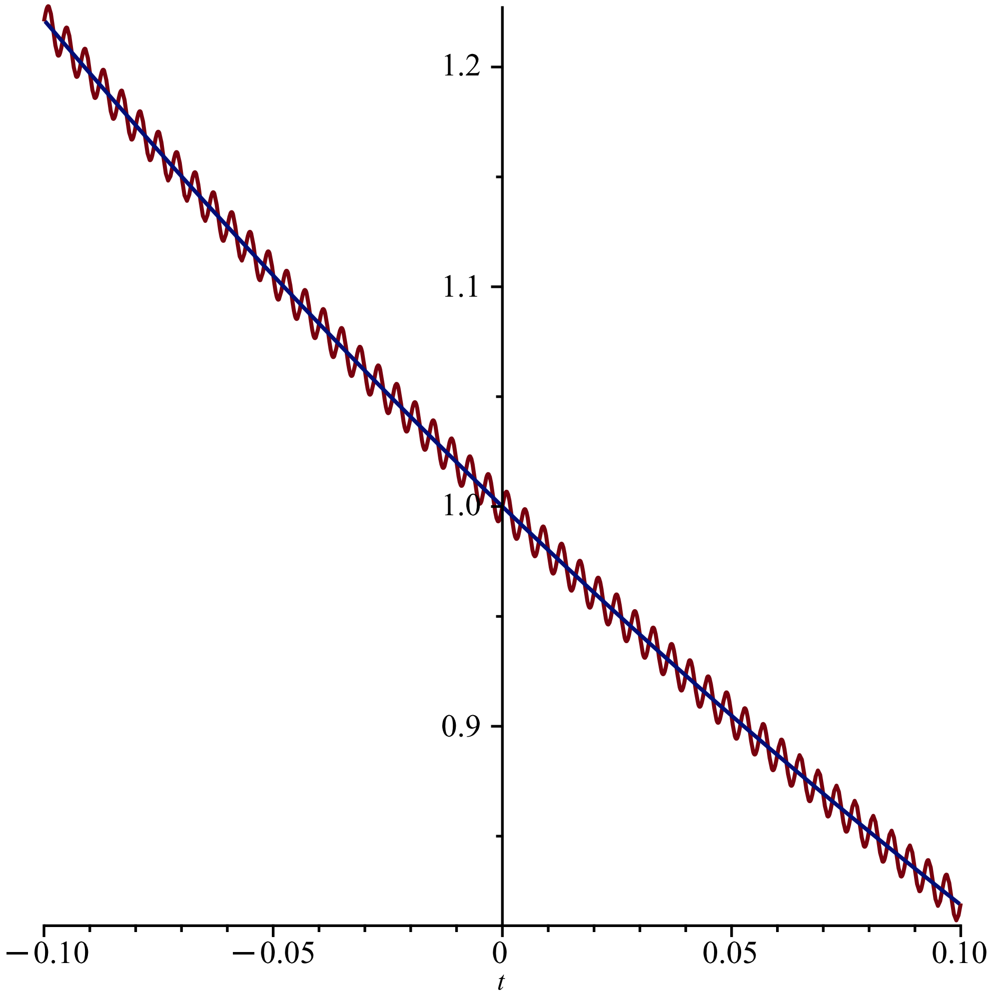
> $plot(\{f, ff\}, t=-1..-0.99)$



> $plot(\{f, ff\}, t=-1..-0.5)$



> $plot(\{f, ff\}, t = -0.1..0.1)$

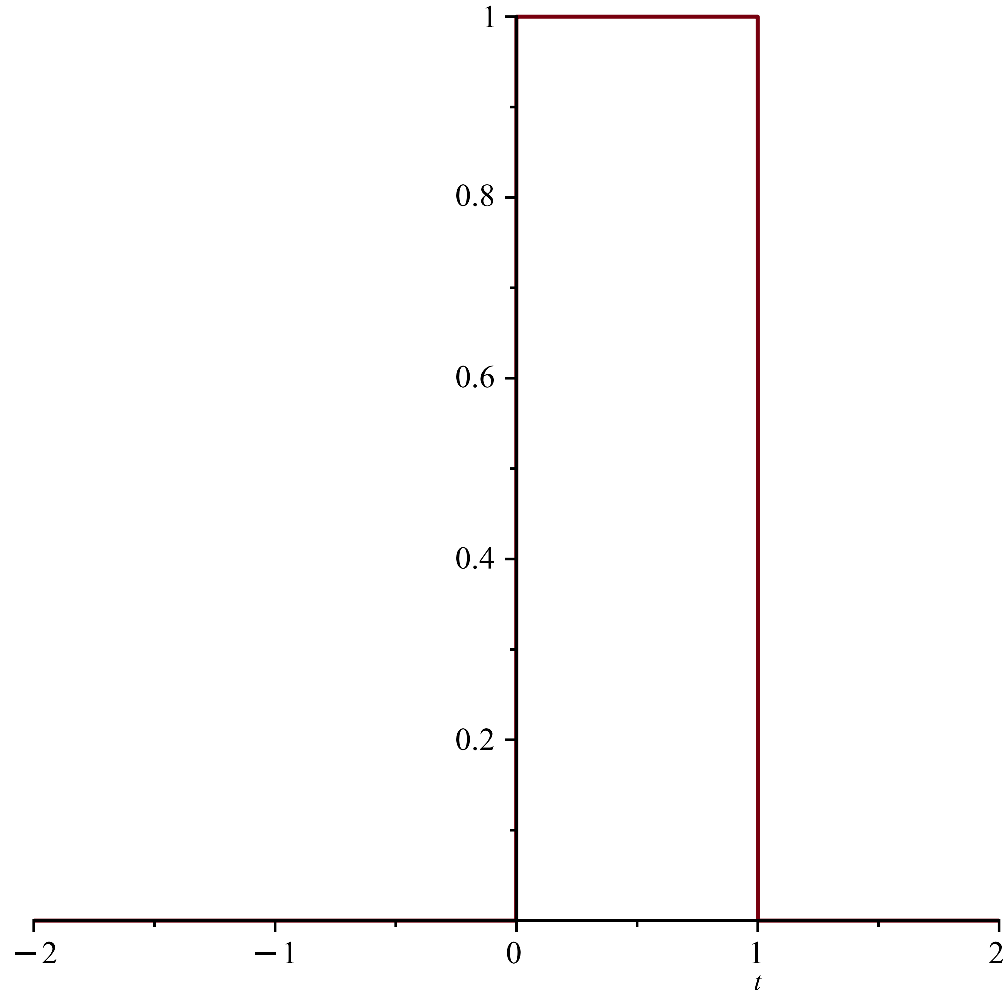


> *restart*

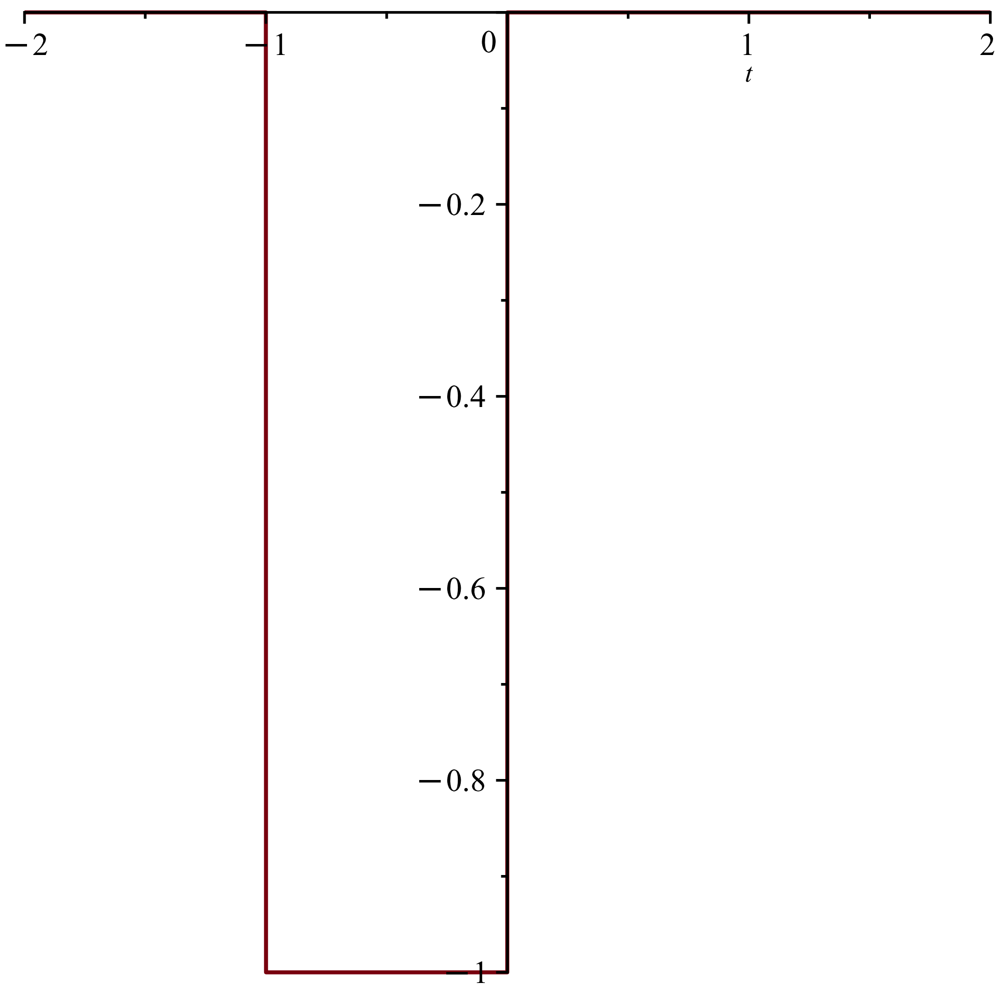
> $g := \text{Heaviside}(t) - \text{Heaviside}(t-1)$

$$g := \text{Heaviside}(t) - \text{Heaviside}(t-1) \tag{37}$$

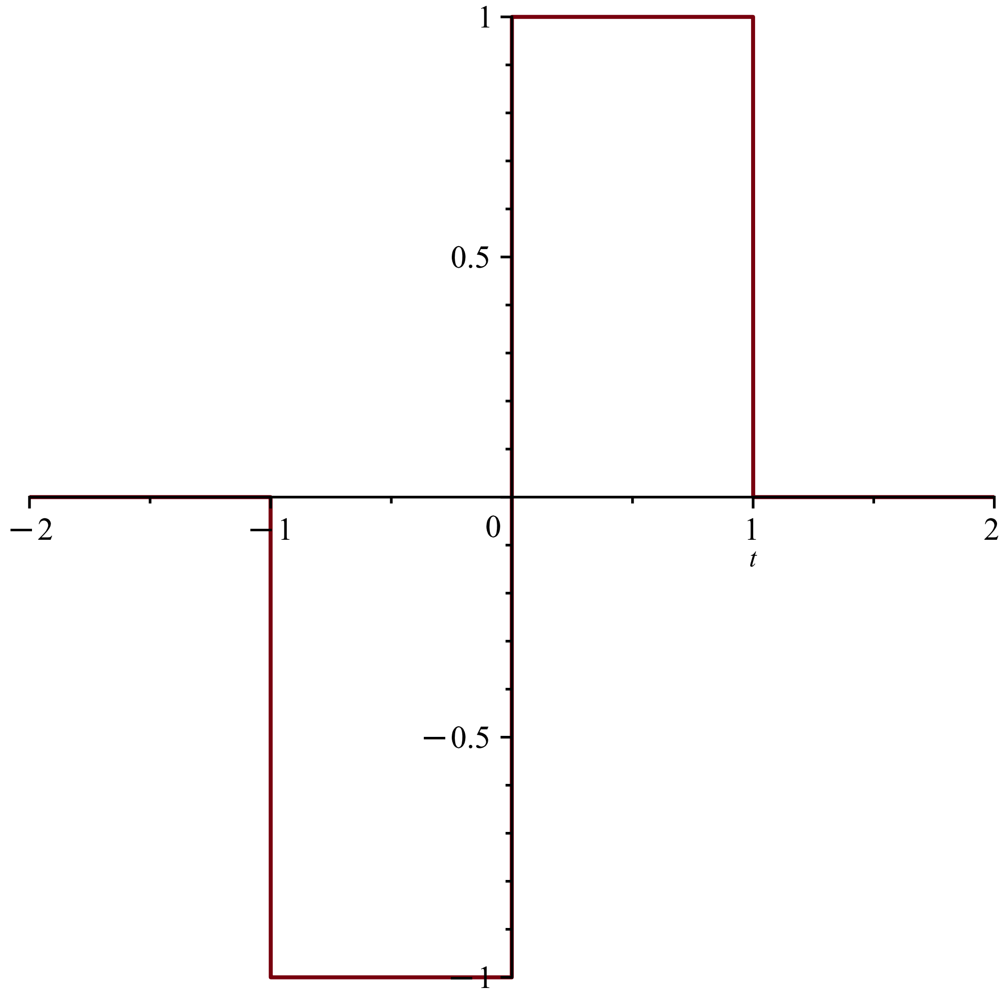
> $plot(g, t = -2 ..2)$



> $h := -\mathrm{Heaviside}(t+1) + \mathrm{Heaviside}(t) : plot(h, t = -2..2)$



> $k := h + g : plot(k, t = -2 .. 2)$



> $L := 1$

$$L := 1 \tag{38}$$

> $a[0] := \frac{1}{L} \cdot int(k, t = -L..L)$

$$a_0 := 0 \tag{39}$$

> $a[n] := \frac{1}{L} \cdot int\left(k \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t = -L..L\right)$

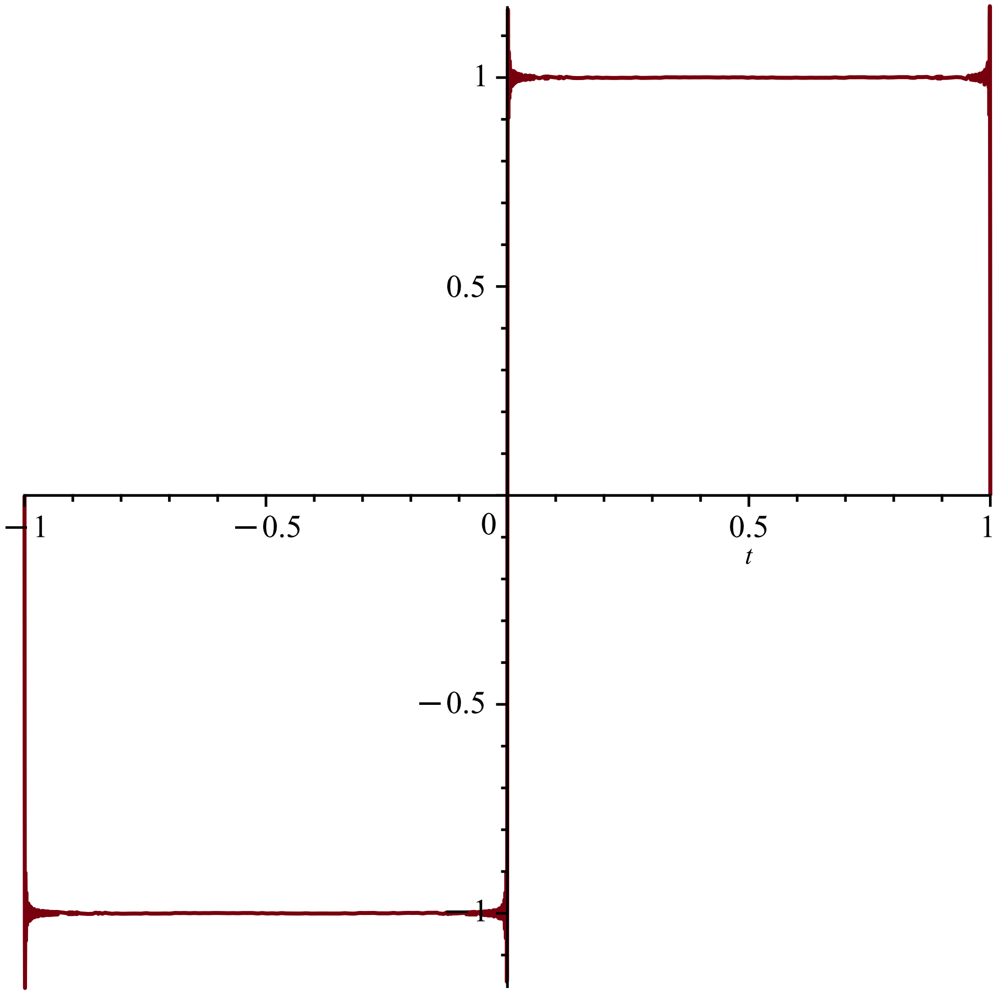
$$a_n := 0 \tag{40}$$

> $b[n] := simplify\left(\frac{1}{L} \cdot int\left(k \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t = -L..L\right)\right)$

$$b_n := \frac{2 - 2\cos(n\,\pi)}{n\,\pi} \tag{41}$$

> $STFk := sum\left(b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), n = 1..1000\right) :$

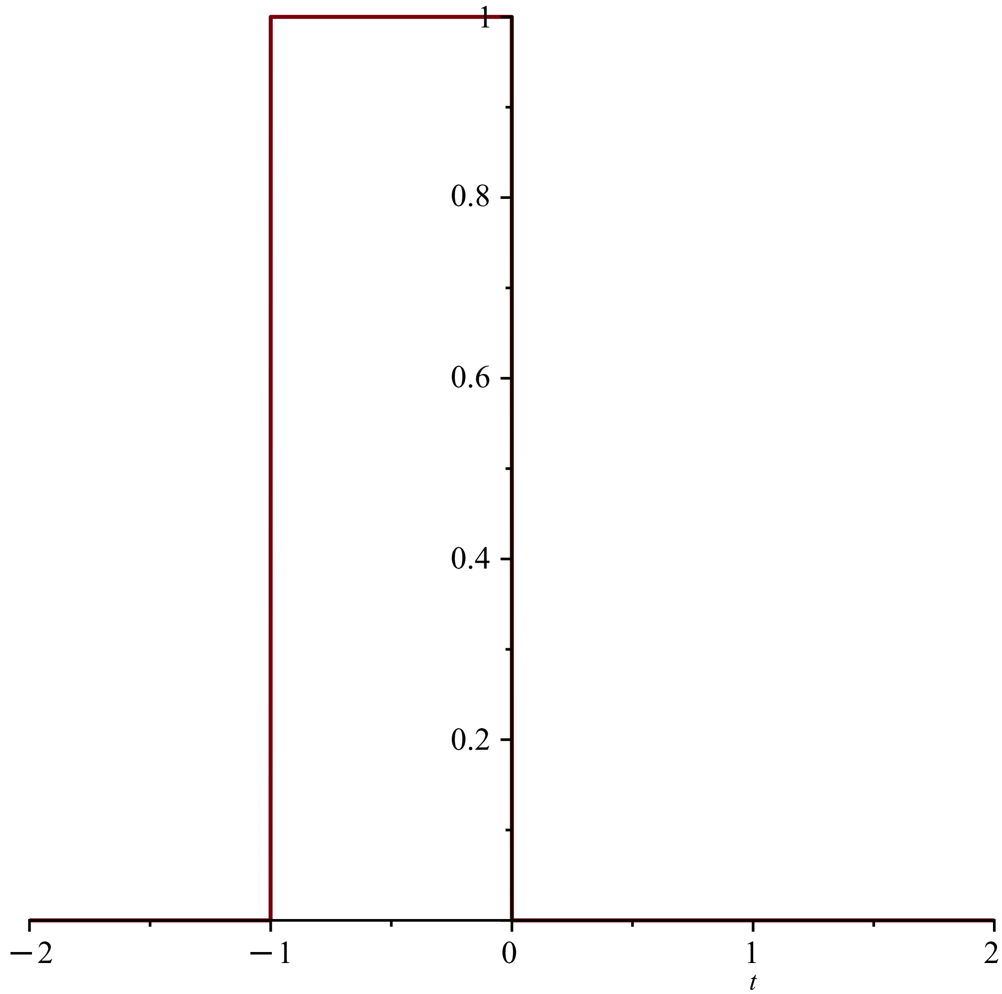
> $plot(STFk, t = -L..L)$



> g

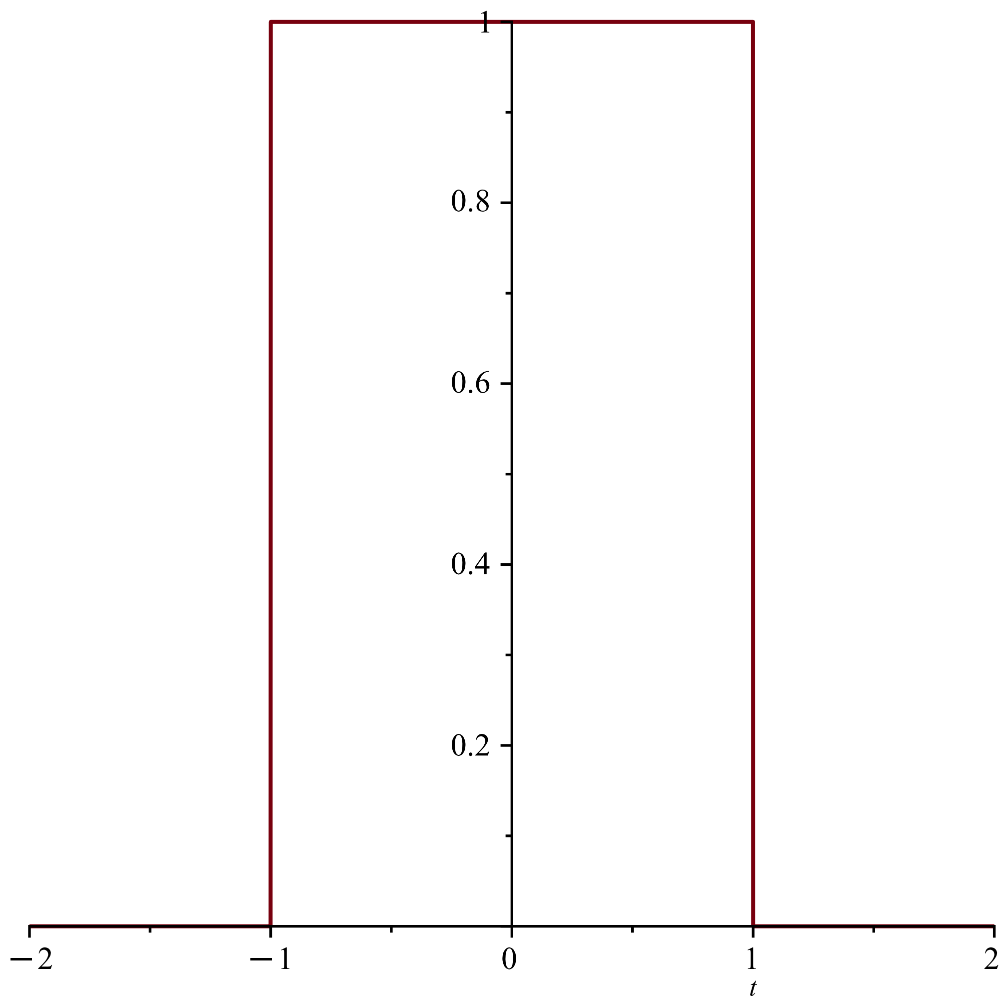
$$\text{Heaviside}(t) - \text{Heaviside}(t-1) \tag{42}$$

> $j := \text{Heaviside}(t+1) - \text{Heaviside}(t) : plot(j, t = -2..2)$



> $l := j + g; plot(l, t = -2..2)$

$$l := \text{Heaviside}(t+1) - \text{Heaviside}(t-1)$$



> $L := 2$

$$L := 2 \tag{43}$$

> $a[0] := \frac{1}{L} \cdot int(l, t = -L..L)$

$$a_0 := 1 \tag{44}$$

> $a[n] := \frac{1}{L} \cdot int\left(l \cdot \cos\left(\frac{n \cdot \text{Pi}}{L} \cdot t\right), t = -L..L\right)$

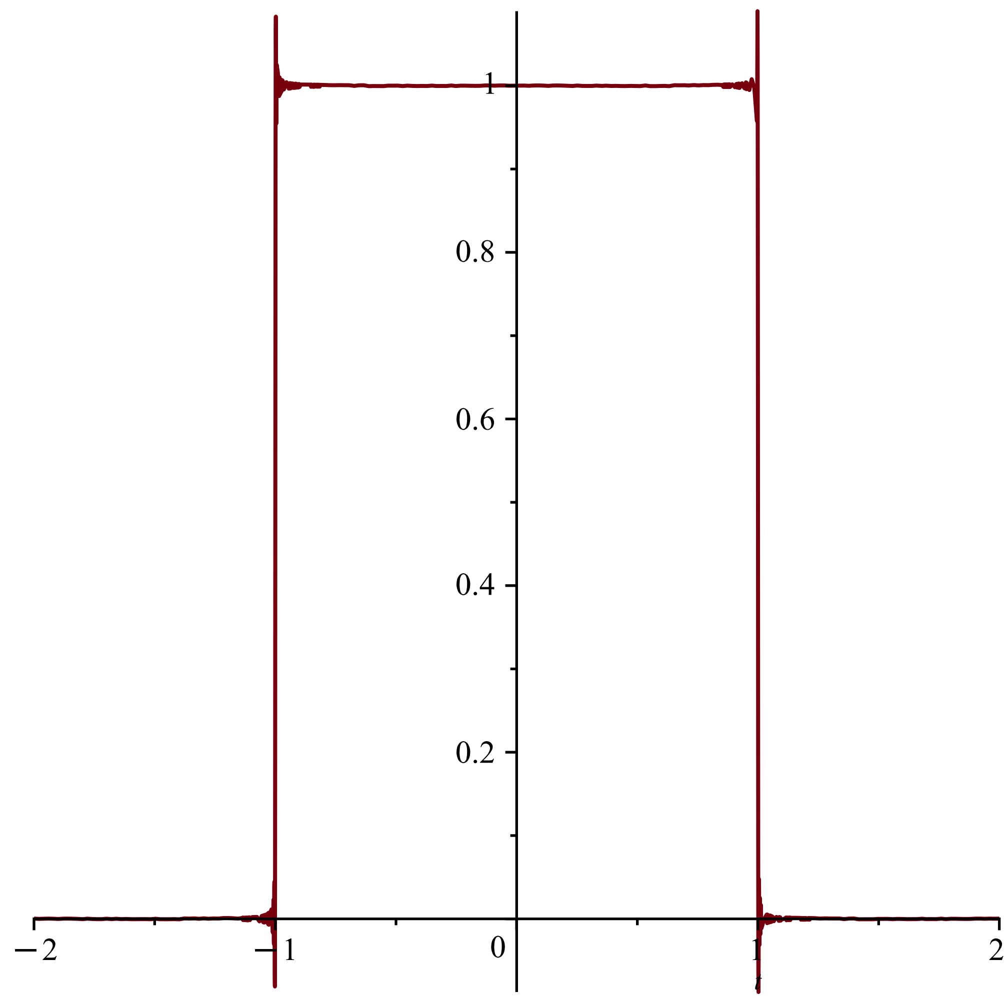
$$a_n := \frac{2 \sin\left(\frac{n \pi}{2}\right)}{n \pi} \tag{45}$$

> $b[n] := \frac{1}{L} \cdot int\left(l \cdot \sin\left(\frac{n \cdot \text{Pi}}{L} \cdot t\right), t = -L..L\right)$

$$b_n := 0 \tag{46}$$

> $STFl := \frac{a[0]}{2} + sum\left(a[n] \cdot \cos\left(\frac{n \cdot \text{Pi}}{L} \cdot t\right), n = 1..1000\right)$:

> $plot(STFl, t = -2..2)$


1
0.8
0.6
0.4
0.2
−2
−1
0
1
2

>